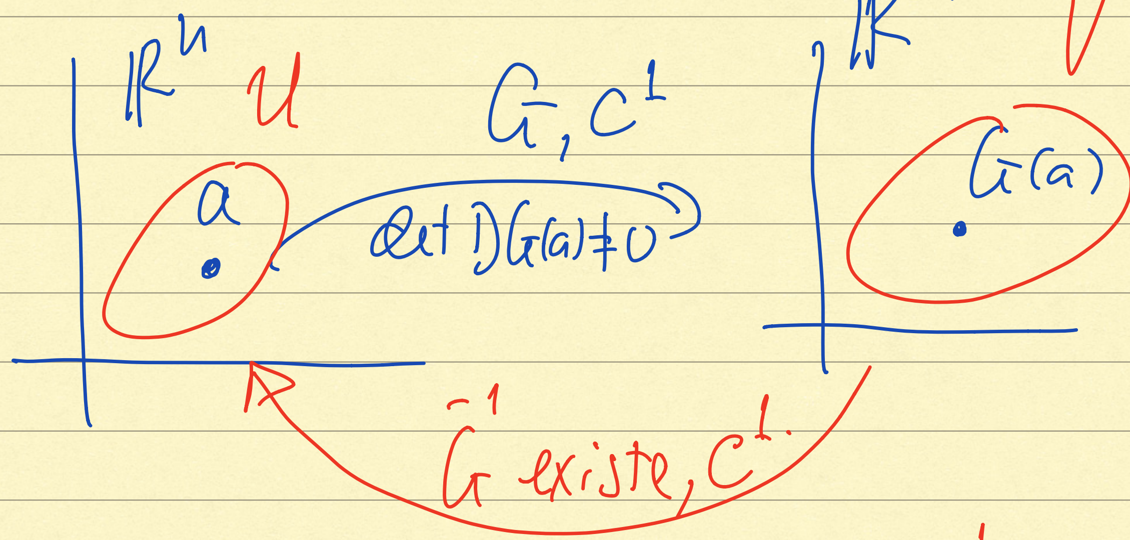
Inversa. Implicita

Inversa: $(\mathbb{R}^n)$



$$DG^{-1}(G(a)) = \left(DG(a)\right)^{-1}$$

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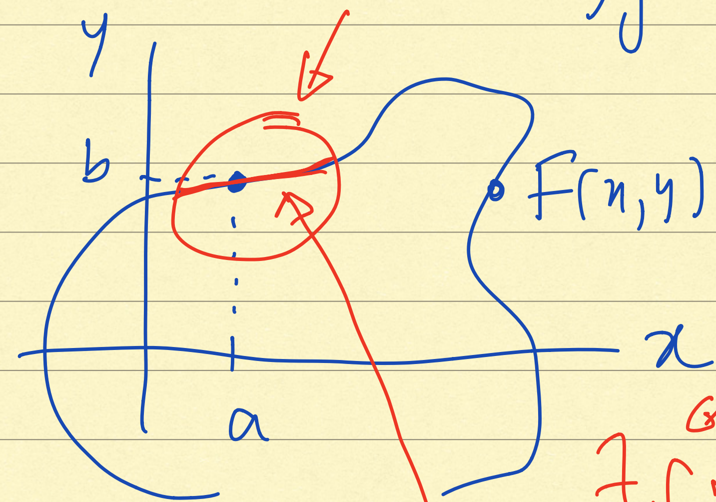
Implicita: $(\mathbb{R}^2)$

1) $F: \mathbb{R}^2 \to \mathbb{R}, \ C^1$

2) $F(a,b)=0$; 3) $\frac{\partial F}{\partial y}(a,b) \neq 0$



$F(x,y)=0$

implícita

$\exists f: \mathbb{R} \to \mathbb{R},\ C^1$:

$F(x,y)=0 \iff y=f(x)$

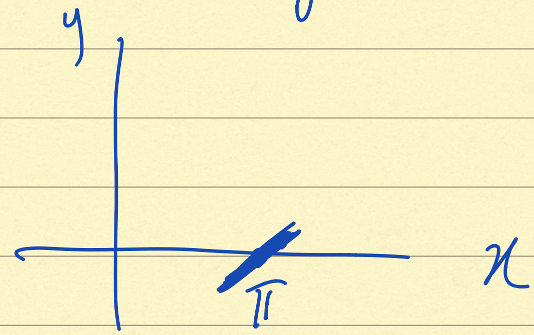
$F(x,f(x))=0$

Cadeia $\Longrightarrow$ $\boxed{\frac{\partial F}{\partial x}(a,b) + \frac{\partial F}{\partial y}(a,b)\, f'(a) = 0}$

Derivada da implícita

Ex: $\operatorname{sen}(x+y) + xy = 0$

$(\pi, 0)$



$F(x,y) = 0 \qquad F: \mathbb{R}^2 \to \mathbb{R},\ C^1$

$F(\pi, 0) = 0$

$$DF(\pi,0) = \begin{bmatrix} \cos(x+y) + y & \cos(x+y) + x \end{bmatrix}_{(\pi,0)}$$

$$= \begin{bmatrix} -1 & \underbrace{-1+\pi}_{\neq 0} \end{bmatrix}$$

$\neq 0 \to \boxed{y = f(x)}$

$$\boxed{f'(\pi)} = - \frac{-1}{\pi - 1} = \frac{1}{\pi - 1} > 0$$

$\underline{\mathbb{R}^3}$: 1)

(1) $F(x,y,z) = 0 \qquad F: \mathbb{R}^3 \to \mathbb{R}, C^1$

(2) $F(a,b,c) = 0$

(3) $DF(a,b,c) = \left[\frac{\partial F}{\partial x}(a,b,c) \quad \frac{\partial F}{\partial y}(a,b,c) \quad \underbrace{\frac{\partial F}{\partial z}(a,b,c)}_{\neq 0} \right]$

(c)

(1), (2) e (3)

$\Downarrow$?

$$\boxed{z = f(x,y) \quad , \quad f: \mathbb{R}^2 \to \mathbb{R}, C^1}$$

$F(x,y,f(x,y)) = 0 \quad \rightarrow$ regra da cadeia.

$$\begin{cases} \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} \frac{\partial f}{\partial y} = 0 \end{cases}$$

(a,b,c)

$$G(x,y,z) = \left(F(x,y,z),\ x,\ y\right)$$

livres

z dependente!

$$G(a,b,c) = (0,\ x,\ y)$$

$$DG(a,b,c) = \begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{(a,b,c)}$$

$\neq 0$

$$\det DG(a,b,c) = \frac{\partial F}{\partial z}(a,b,c) \neq 0\ !$$

$\Rightarrow G^{-1}$ existe, C^1.

$$G(x,y,z) = (0,x,y)$$

$$\underline{(x,y,z)} = G^{-1}(0,x,y)$$

$z = f(x,y)$, $f: \mathbb{R}^2 \to \mathbb{R}$, C^1

implícita

Ex:

$2x + 3y + \boxed{5}z = 4$ A.L. (contas!)

$\neq 0$ linear!

$$\begin{bmatrix} 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [4] = 4$$

$$z = \frac{4}{5} - \frac{2}{5}x - \frac{3}{5}y = f(x,y).$$

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2) $\begin{cases} F_1(x,y,z) = 0 \\ F_2(x,y,z) = 0 \end{cases}$ 2 equações
3 variáveis

$F = (F_1, F_2)$ $\qquad$ $F: \mathbb{R}^3 \to \mathbb{R}^2$, C^1

(a,b,c) $\qquad$ $F(a,b,c) = (0,0)$

$$DF(a,b,c) = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \underbrace{\begin{matrix} \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \end{matrix}}_{} \\ \frac{\partial F_2}{\partial x} & \end{bmatrix}_{(a,b,c)}$$

D

<u>linear</u>: $\begin{cases} a_1 x + b_1 y + c_1 z = d_1 \\ a_2 x + b_2 y + c_2 z = d_2 \end{cases}$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$\underbrace{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}}_{\det \neq 0} \implies \begin{cases} y = y(x) \\ z = z(x) \end{cases}$$

$$\begin{cases} 2x + 3y + z = 0 \\ 4x + 5y - z = 1 \end{cases}$$

$$\begin{bmatrix} 2 & \underbrace{\boxed{\begin{matrix} 3 & 1 \\ 5 & -1 \end{matrix}}}_{-8 \neq 0} \\ 4 & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{cases} 3y + z = -2x \\ 5y - z = 1 - 4x \end{cases}$$

$$\underbrace{\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}}_{A} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -2x \\ 1-4x \end{bmatrix} \qquad \det A \neq 0$$

$$\begin{pmatrix} y \\ z \end{pmatrix} = A^{-1} \begin{bmatrix} -2x \\ 1-4x \end{bmatrix}$$

1) $F: \mathbb{R}^3 \to \mathbb{R}^2$, C^1

2) $F(a,b,c) = (0,0)$

3) $\det D_{y,z} F(a,b,c) = \begin{bmatrix} \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \end{bmatrix}_{(a,b,c)} \neq 0$

$\Longrightarrow \begin{cases} y = y(x) \\ z = z(x) \end{cases}$, C^1 implícitas

Regra de cadeia:

$$\begin{cases} F_1(x, y(x), z(x)) = 0 \\ F_2(x, y(x), z(x)) = 0 \end{cases}$$

$$\boxed{\begin{cases} \dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_1}{\partial y} y' + \dfrac{\partial F_1}{\partial z} z' = 0 \\ \dfrac{\partial F_2}{\partial x} + \dfrac{\partial F_2}{\partial y} y' + \dfrac{\partial F_2}{\partial z} z' = 0 \end{cases}}$$

linear

$$\begin{bmatrix} \dfrac{\partial F_1}{\partial x} & \dfrac{\partial F_1}{\partial y} & \dfrac{\partial F_1}{\partial z} \\[2ex] \dfrac{\partial F_2}{\partial x} & \dfrac{\partial F_2}{\partial y} & \dfrac{\partial F_2}{\partial z} \end{bmatrix}_{(a,b,c)}$$

$$\det \neq 0 \implies \begin{array}{l} y = y(x) \\ z = z(x) \\ C^1. \end{array}$$

<u>Exemplo</u>: $\begin{cases} \operatorname{sen}(x+y) + xy = 0 \\ x + z = \pi \end{cases}$ ✓

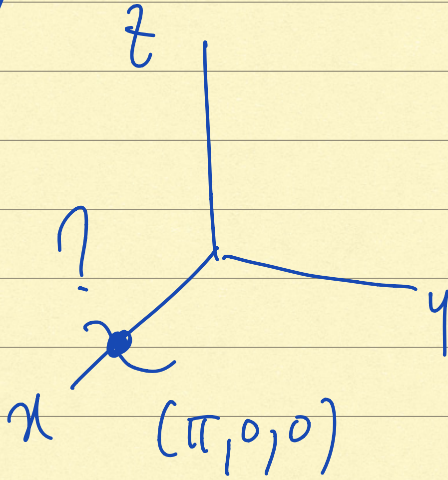
$(\pi, 0, 0)$ ✓

$$\begin{bmatrix} \cos(x+y) + y & \cos(x+y) + x & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$
= \begin{array}{ccc} b' & dy' & dz' \\ \end{array}
$$

$$
= \begin{bmatrix} -1 & -1+\pi & 0 \\ 1 & 0 & 1 \end{bmatrix}
$$

$$
\underbrace{\begin{vmatrix} -1+\pi & 0 \\ 0 & 1 \end{vmatrix}}_{\det \neq 0} \Rightarrow \quad y = y(x), \quad z = z(x), \quad C^1
$$



$$0 = y(\pi)$$

$$0 = z(\pi)$$

$$
\begin{cases} -1 + (\pi - 1)\, y'(\pi) + \cancel{0 \times z'(\pi)} = 0 \\ 1 + \cancel{0 \times y'(\pi)} + z'(\pi) = 0 \end{cases}
$$

$$
\boxed{y'(\pi) = \frac{1}{\pi - 1} \; ; \; z'(\pi) = -1}
$$